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METHOD AND APPARATUS FOR MAPPING UNCERTAINTY AND GENERATING A MAP OR A CUBE BASED ON CONDITIONAL SIMULATION OF RANDOM VARIABLES

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CROSS REFERENCE TO RELATED APPLICATIONS

This application is a Utility application of prior pending provisional application serial

15   number 60/135,904, filed 05/25/99, and entitled "Uncertainty Mapping".

BACKGROUND OF THE INVENTION

The subject matter of the present invention relates to a software based method and

20   associated apparatus for mapping uncertainty by producing one or more "maps", such

as a probability map or a cutoff map or a confidence limit map, and/or one or more

cubes, based on the conditional simulation of a set of random variables.

The estimation at any spatial location (x, y, z) of the value of a parameter

25   (i.e., a random variable), such as porosity or permeability, from a set of scattered

observations of data representing such a parameter may be achieved by a method

known as "Kriging". For a reference which describes "Kriging", refer to either of the

following two references: (1) Journel, A.G. "Fundamentals of Geostatistics in Five

Lessons", Short course in Geology, vol 8, 44 pp, AGU, Washington, D.C. 1989, or

30   (2) Deutsch, Clayton V. and Andre Journel, "GSLIB Geostatistical Software Library

and User's Guide second edition", Oxford University Press, New York, Oxford,

1998; the disclosures in each of the above two references which discuss "Kriging"

are incorporated by reference into this specification. For example, if a set of scattered data samples represents porosity (obtained, for example, by mapping a cross section of an earth formation through which a plurality of wellbores are drilled as indicated in figures 4 and 5), when the cross section is gridded, the "Kriging" method

5   can determine, at each intersection of the grid, the expected (mean) value of porosity and its standard deviation.

The result of estimating this value at a regular grid results in a smooth surface of the expected value generally following the data (see figures 4 through 16). If the

10   principle of exactitude is applied, this surface will actually go through the data if the data falls on a grid location. The error variance is also calculated at each grid location. At the data locations, this error variance equals zero unless the observation error variance is included in the calculation, in which case, this is the error variance at the data locations.

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It is common practice to render a more realistic estimate of the variable by randomizing the answer using the estimated error variance. This is called a 'conditional simulation' because it is conditioned by the data and produces one out of any number of possible realizations. It is then customary to assess the risk associated

20   with exploitation of the estimated accumulation of ore, hydrocarbons or other valuable commodities by sampling a good many of the realizations and ranking them in low, medium, and high ranges, according to the economic value of the estimated deposits. This is a lengthy and costly procedure.

25   Therefore, a new method of mapping uncertainty is needed.

A method of mapping an earth formation has been disclosed in U.S. Patents 5,563,949 and 5,995,907. Another method for mapping an earth formation and generating a cube which contains a plurality of such maps is disclosed in prior

30   pending application serial number 09/377,573, filed 08/19/99, and entitled "Seismic signal processing method and apparatus for generating a cube of variance values".

SUMMARY OF THE INVENTION

Accordingly, it is a primary object of the present invention to provide a new method for mapping uncertainty, the new method being based on the conditional simulation of random variables.

In accordance with the above object of the present invention, it is a primary feature of the present invention to generate one or more maps representing the characteristics at each intersection of a gridded cross-section, where the gridded cross-section represents a time slice or a horizon through an earth formation, in response to a plurality of scattered data samples on such cross-section, where the data samples represent a plurality of parameters located at a plurality of random locations on the cross section, by performing the steps of: (a) obtaining a unique cumulative distribution function associated with each intersection of the gridded cross-section in the earth formation, (b) choosing a value from each of the cumulative distribution functions at each of the intersections of the gridded cross section, and (c) assigning such value to its respective intersection of the gridded cross section and assigning a unique color to each such value, thereby generating a map illustrating the characteristics of the cross section through the earth formation.

It is a further feature of the present invention to utilize the above referenced method for generating one or more maps to also generate a cube, the cube including a plurality of such maps which illustrate a set of characteristics associated with a plurality of cross sections through an earth formation.

It is a further feature of the present invention to obtain the unique cumulative distribution function associated with each intersection of the gridded cross-section by first obtaining a unique probability density function associated each such intersection, and then obtaining the unique cumulative distribution function from the probability

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density function associated with each intersection of the gridded cross section through the earth formation.

It is a further feature of the present invention to generate one or more maps
5   representing the characteristics at each intersection of a gridded cross section representing a time slice or a horizon in an earth formation in response to a plurality of scattered data samples which represent a plurality of parameters located at a plurality of random locations on said cross section by performing the steps of:
(a) gridding the cross section thereby creating a plurality of intersections on said cross
10   section, (b) Kriging the gridded cross section thereby producing a plurality of expected (mean) values of the parameter and a corresponding plurality of standard deviations associated, respectively, with the plurality of intersections on the gridded cross section in response to the plurality of scattered data samples, (c) generating a plurality of probability density functions corresponding, respectively, to the plurality
15   of expected values and standard deviations which are associated, respectively, with the plurality of intersections of the gridded cross section, a unique probability density function being associated with each intersection of the gridded cross section,
(d) integrating each of the probability density functions at each of the intersections of the gridded cross section to generate a plurality of cumulative distribution functions
20   associated, respectively, with the plurality of intersections, a unique cumulative distribution function being associated with each intersection of the gridded cross section, (e) choosing a value from each of the cumulative distribution functions at each of the intersections of the gridded cross section, and (f) assigning such value to its associated intersection of the gridded cross section and assigning a unique color to
25   each such value, thereby generating a map illustrating the characteristics of the cross section (which could represent a time slice or a horizon) through the earth formation.

It is a further feature of the present invention to generate one or more maps representing the characteristics at each intersection of a cross-section in an earth
30   formation in response to a plurality of scattered data samples, wherein the choosing step (e) for choosing a value from each of the cumulative distribution functions

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(CDF) at each of the intersections of the gridded cross section comprises the step of selecting a cutoff value of a parameter on the x-axis of a CDF and choosing the associated value (Pcu) on the y-axis of the CDF, one minus such value [(1-Pcu)] representing the probability that the parameter is greater than the cutoff.

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It is a further feature of the present invention to generate one or more maps representing the characteristics at each intersection of a cross-section in an earth formation in response to a plurality of scattered data samples, wherein the choosing step (e) for choosing a value from each of the cumulative distribution functions

10   (CDF) at each of the intersections of the gridded cross section comprises the step of selecting a value of probability on the y-axis of the CDF and choosing its associated value "Xp" of a parameter on the x-axis of the CDF.

It is a further feature of the present invention to generate one or more maps representing the characteristics at each intersection of a cross-section in an earth

15   formation in response to a plurality of scattered data samples, wherein the choosing step (e) for choosing a value from each of the cumulative distribution functions (CDF) at each of the intersections of the gridded cross section comprises the step of selecting a confidence range (e.g., 90%) on the y-axis of the CDF and choosing one of

20   three values on the x-axis of the CDF: a lower limit, a spread, and an upper limit of such parameter on the x-axis of the CDF.

It is a further feature of the present invention to generate one or more maps representing the characteristics at each intersection of a cross-section in an earth

25   formation in response to a plurality of scattered data samples, wherein the value selected via the choosing step (e) undergoes an affine correction, the value assigned to its associated intersection of the gridded cross section via the assigning step (f) being a "corrected value" which underwent a correction via the affine correction.

30   It is a further feature of the present invention to generate a cube representing a cubic volume through an earth formation which includes a plurality of gridded cross

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sections, where each gridded cross section includes a plurality of intersections, by performing the step of assigning the aforementioned "corrected value" to each intersection of each gridded cross section through a cubic volume of the earth formation.

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In accordance with this object and other objects and features of the present invention, a new method is presented to produce probability, cutoff, confidence limit and connectivity maps in one single operation. In addition, the new method also produces a cube containing a plurality of said maps, where each of the maps in the

10   cube may represent the probability map, the cutoff map, or the confidence limit map. This is accomplished by using the estimated gaussian distribution at each intersection of the grid given by an expected value and an error variance. For computations on non-gaussian problems, the data may be converted to gaussian by using the normal score transform [refer to the following reference: Deutsch, Clayton V. and Andre

15   Journel, "GSLIB Geostatistical Software Library and User's Guide second edition", Oxford University Press, New York, Oxford, 1998, page 141, the disclosure of which is incorporated by reference into this specification]. The answers may be converted back to the prior distribution by a back transform.

20   More particularly, given a cross section through an earth formation, which cross section could represent a time slice or horizon in the earth formation, when a plurality of the random variables or parameters are associated with the cross section (for example, porosity or permeability), one or more maps can be generated, where each map illustrates a set of characteristics of that cross section, and a cube can also be

25   generated, where the cube includes a plurality of such maps corresponding to a plurality of the cross sections through the earth formation. This can be accomplished by: (1) gridding the cross section thereby producing a gridded cross section including the plurality of associated random variables, the gridded cross section including a plurality of intersections, (2) using the random variables or parameters (such as

30   porosity) on the cross section, obtaining a corresponding plurality of parameters at each of the intersections of the gridded cross section by "Kriging" the plurality of

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random variables on the gridded cross section, the plurality of parameters at each intersection of the gridded cross section including a unique expected value of the parameter (i.e., the mean value) and a unique standard deviation, the unique expected value and the unique standard deviation at each intersection of the gridded cross

5   section corresponding to a unique probability density function at each intersection, (3) integrating the probability density function at each intersection of the gridded cross section to obtain a unique cumulative distribution function at each intersection of the gridded cross section, (4) from the unique cumulative distribution function at each intersection of the gridded cross section, obtaining a value from that cumulative

10   distribution function, a plurality of values being obtained which are associated, respectively, with the plurality of intersections of the gridded cross section, (5) correcting the plurality of values obtained from the corresponding plurality of cumulative distribution functions and associated, respectively, with the plurality of intersections by using an "affine correction" thereby producing a plurality of

15   'corrected values' corresponding, respectively, to the plurality of intersections of the gridded cross section, and (6) assigning each of the 'corrected values' to a particular intersection of the gridded cross section, and assigning a unique color to each such corrected value.

20   The above step (4), for obtaining a value from the cumulative distribution function, would include: (4a) selecting a cutoff on the x-axis of the cumulative distribution function and determining a probability "Pcu" on the y-axis, (1-Pcu) being the 'value' which represents the probability that the parameter is greater than the cutoff, or (4b) selecting a probability on the y-axis of the cumulative distribution function and

25   determining a cutoff "Xp" on the x-axis of the cumulative distribution function, "Xp" being the 'value' which represents a cutoff, or (4c) selecting a confidence limit range on the y-axis of the cumulative distribution function, and choosing three 'values' from the x-axis of the cumulative distribution function: the "lower limit", the "spread", and the "upper limit".

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A lower cutoff map is generated by plugging in the desired probability and measuring the quantity via the local cumulative distribution function (cdf). In this manner, we produce a map of lower cutoff values for the desired probability. We can then say that, at any given (x, y, z) location, the content is at least the cutoff value with a

5   probability of, for example, ninety percent. A lower cutoff cube can be generated from a plurality of the lower cutoff maps. A probability map is generated by plugging in the desired lower cutoff and evaluating the probability at any (x, y, z) location, again using the gaussian cumulative distribution function (cdf). A probability cube can be generated from a plurality of the probability maps.

10   Confidence limit maps are obtained by plugging in half the compliment of the confidence level and one minus this measure to the lower and upper limit of the cumulative distribution function (cdf). The difference between those limits is the data range for the given confidence level. For example, a confidence level of 90% would correspond to .05 and .95 on the cdf. A confidence limit cube can also be generated

15   from a plurality of the confidence limit maps. To compensate for the smoothing effect of randomizing within the estimated error variance, an affine correction is applied. This is accomplished by co-generating one realization without feeding the newly estimated nodes back into the simulation. The mean and the standard deviation of this realization are then calculated and compared with the mean and the standard

20   deviation of the data. The correction ensures that the resulting maps are derived from the same global mean and standard deviation as the original data. A connectivity map may be produced from a permeability cutoff map for a given probability by superimposing an arbitrary bound above which the formation is said to be permeable and below which it is impermeable. We may use indicators such as a "one" to denote

25   permeability and a "zero" to denote impermeability. In this fashion, a reservoir may be divided into several unconnected compartments, which will influence the economics of the extraction of hydrocarbons. Permeability cutoff maps may be submitted to a fluid flow simulator to estimate the minimum production for a given acceptable probability or risk. It should be remembered, however, that, although this

30   is a lower cutoff surface, it is smooth and does not represent the natural irregularities we expect in the reservoir.

Further scope of applicability of the present invention will become apparent from the detailed description presented hereinafter. It should be understood, however, that the detailed description and the specific examples, while representing a preferred

5   embodiment of the present invention, are given by way of illustration only, since various changes and modifications within the spirit and scope of the invention will become obvious to one skilled in the art from a reading of the following detailed description.

10   BRIEF DESCRIPTION OF THE DRAWINGS

A full understanding of the present invention will be obtained from the detailed description of the preferred embodiment presented hereinbelow, and the accompanying drawings, which are given by way of illustration only and are not

15   intended to be limitative of the present invention, and wherein:

Figure 1 illustrates a workstation or other computer system which stores a novel "uncertainty mapping" software which is processed by a processor in response to input data comprised of randomly scattered data samples;

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Figures 2 and 3 illustrates a flowchart used for discussion of the production of the Probability maps by the uncertainty mapping software of figure 1;

Figures 4 through 16 illustrates several figures which are used during the discussion

25   of the production of the Probability maps by the uncertainty mapping software of figure 1;

Figures 17 and 18 illustrate figures which are used during the discussion of the production of the cutoff map by the uncertainty mapping software of figure 1;

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Figures 19 through 22 illustrate figures which are used during the discussion of the production of the confidence limit map by the uncertainty mapping software of figure 1;

5   Figure 23 illustrates the affine correction applied to the (1-Pcu), the Xp, the lower limit, the spread, and the upper limit;

Figures 24 through 28 illustrate the corrected values of (1-Pcu), Xp, lower limit, spread, and upper limit, corrected by the affine correction, which are assigned to and

10   plotted on a gridded cross section through an earth formation;

Figures 29 through 32 illustrates the function of the affine correction;

Figures 33 and 34 illustrates different examples of probability maps which are

15   generated by the uncertainty mapping software of the present invention;

Figure 35 illustrates Kriging with exactitude, including estimated value (fig 35a), error variance and without exactitude (fig 35b), expected value (fig 35c), and error variance of the expected value (fig 35d);

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Figures 36 through 43 illustrate how the methods discussed in this specification in connection with figures 1 through 35, for generating a probability map and a lower cutoff map and a confidence limit map, may be used for generating a cube which represents a cubic volume of an earth formation and illustrates a set of characteristics

25   of the earth formation within that cube; and

Figures 36 through 43 illustrate how a cube is generated given the methods discussed with reference to figures 1 through 35 for determining a 'corrected value' to assign to each intersection of a gridded cross section.

DESCRIPTION OF THE PREFERRED EMBODIMENT

In the prior art involving Conditional Simulation, the estimation at any spatial

5   location (x,y,z) of the value of a gaussian random variable from a set of scattered data

samples is accomplished by a method known as "Kriging" [as previously mentioned,

refer to either: (1) Journel, A.G. "Fundamentals of Geostatistics in Five Lessons",

Short course in Geology, vol 8, 44 pp, AGU, Washington, D.C. 1989, or (2) Deutsch,

Clayton V. and Andre Journel, "GSLIB Geostatistical Software Library and User's

10   Guide second edition", Oxford University Press, New York, Oxford, 1998]. The

result of estimating this value at a regular grid results in a smooth surface of the

expected value generally following the data. If the principle of exactitude is applied,

this surface will actually go through the data if the data falls on a grid location. The

error variance is also calculated at each grid location. At the data locations, this error

15   variance equals zero unless the observation error variance is included in the

calculation, in which case, this is the error variance at the data locations. It is

common practice to render a more realistic estimate of the variable by randomizing

the answer using the estimated error variance. This is called a 'conditional

simulation' because it is conditioned by the data and produces one out of any number

20   of possible realizations. For a teaching on 'conditional simulation', refer to:

"Fundamentals of Geostatistics in Five Lessons", by A. G. Journel, Short course in

Geology, vol 8, 40 pp, AGU, Washington, D.C. 1989.

For example, in order to "Krige the expected value and standard deviation", consider

25   the following equations [refer to reference (2): Deutsch, Clayton V. and Andre

Journel, "GSLIB Geostatistical Software Library and User's Guide second edition",

Oxford University Press, New York, Oxford, 1998, page 14, equations II.12 and

II.13]:

30   $$Z_{SK}^{*}(u) = \Sigma \lambda_{\alpha}(u) Z(u_{\alpha}) + \left(1 - \Sigma \lambda_{\alpha}(u)\right) m \qquad (1)$$

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The corresponding minimized estimation variance is:

$$\sigma_{SK}{}^2(u) = C(0) - \Sigma\lambda_\alpha(u)C(u-u_\alpha) \tag{2}$$

For ordinary "Kriging", the sum of the set of weights is constrained to unity, which leads to:

5 $$Z_{OK}{}^*(u) = \Sigma\lambda_\alpha(u)Z(u_\alpha) \tag{3}$$

where:

$Z_{SK}{}^*(u)$ = estimate of unsampled data value at location 'u' using "simple kriging",

$\lambda_\alpha(u)$ = weights determined to minimize the estimation variance,

$Z(u_\alpha)$ = neighboring data values used in the estimate where $\alpha = 1,....,n$,

10 m = the a priori known stationary mean of the data, and

$C(u-u_\alpha)$ = covariance.

The "Kriging" estimate is also known as the 'expected value' or the 'local mean' of the estimated distribution of the data, and the estimation variance as the squared

15 standard deviation of the distribution:

$$\mu_1 = \Sigma\lambda_\alpha(u)Z(u_\alpha) \tag{4}$$

Using the mean and the standard deviation, we may now execute a conditional stochastic simulation as follows:

$$xp = F^{-1}\{rand[p(0|1)]\} \tag{5}$$

20 $$\xi = xp \bullet \sigma_1 + \mu_1 \tag{6}$$

where:

F^{-1} = inverse gaussian transform,

μ_1 = estimated local mean,

σ_1 = estimated local standard deviation,

25 xp = data value corresponding to p,

p   = randomized probability, and

ξ = estimated stochastic variable.

In accordance with the present invention, a new method and associated apparatus will
generate probability maps, cutoff maps, and confidence limit maps in one single
operation. In addition, in accordance with the present invention, given the novel
methods of the present invention for generating the probability maps, cutoff maps,
and confidence limit maps, the new method and apparatus of the present invention
will also generate a cube which illustrates a set of earth formation characteristics
disposed within a cubic volume of an earth formation, the cube including a plurality
of gridded cross sections, the grid of each cross section including a plurality of
intersections, the novel method of the present invention generating a 'corrected value'
for each intersection of each grid of each cross section in the cube.

Referring to figure 1, a computer system (e.g., a workstation computer system) is
illustrated and it includes a processor 10 connected to a system bus 12, a recorder or
display device 14 connected to the system bus 12, a memory or other storage device
(e.g., a computer readable medium) 16 connected to the system bus 12, and a source
of input data 18 comprised of a set of randomly scattered data samples 18, such as the
randomly scattered data samples shown in figures 4, 5, and 6. A CD-Rom 20 stores a
novel software known as the "uncertainty mapping software", the uncertainty
mapping software on the CD-Rom 20 being loaded into the computer system and
stored in the memory 16 of the computer system. Now that the software is loaded
into the computer system, the "uncertainty mapping software" will hereinafter be
referred to as the "uncertainty mapping software 16".

The computer system of figure 1 may be a personal computer (PC), a workstation, a
mainframe, etc. Examples of possible workstations include a Silicon Graphics Indigo
2 workstation and a Sun SPARC workstation. The processor 10 of the computer

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system may be, for example, a microprocessor, microcontroller, or a mainframe or workstation processor. The memory 16 may be, for example, a hard disk, ROM, CD-Rom, DRAM, or other RAM, flash memory, magnetic storage, optical storage, registers, or other volatile and/or non-volatile memory.

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When the uncertainty mapping software 16 of the present invention is executed by the processor 10 of figure 1, the processor 10 will generate and display (via the recorder or display device 14 of figure 1) three different types of maps: (1) Probability maps, (2) Lower cutoff maps, and (3) Confidence limit maps (also called "confidence maps)

10   in one single operation. In addition, the processor 10 will generate and display the cube which illustrates the set of earth formation characteristics disposed within a cubic volume of an earth formation. The cube and each of the three different types of maps will be discussed in the following paragraphs.

15   Referring to figures 2 and 3, when the uncertainty mapping software 16 of figure 1 produces the cube and the Probability maps and the lower cutoff maps and the confidence limit maps, the computer system of figure 1 executes the uncertainty mapping software 16 of the present invention. When executed, the uncertainty mapping software 16 performs a new method which uses the estimated gaussian

20   distribution at each grid location (intersection) given by the expected value and the error variance. When the Uncertainty Mapping Software 16 is executed by the processor 10 of figure 1, the new method that is performed for generating a new and novel "cube", as well as the "probability map", "lower cutoff map" and "confidence limit map", is illustrated in the flowchart/block diagram shown in figures 2 and 3.

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The novel method of the present invention for generating the maps will be first discussed. When these novel methods for generating the maps is understood, the novel method for generating the cube will be discussed.

30   In figures 2 and 3, referring initially to figure 2, each probability value ["(1-Pcu)"] on the "Probability map", and each lower cutoff "Xp" on the "lower cutoff map", and

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each "lower limit, spread, and upper limit" on each of three types of "confidence limit maps" (located at each intersection of a grid imposed on the map) is determined by performing the following method steps:

5   (1) Using a horizontal slice through a predetermined volume of earth including a plurality of wellbores, generate a cross section which includes a plurality of parameter data, such as porosity, block 22;

     (2) grid the cross section, block 24;

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     (3) Provide the gridded cross section including the plurality of parameter data representing scattered data samples 44a, 44b, 44c, and 44d as input data to a Kriging algorithm, execute the Kriging algorithm using that input data, and generate a second gridded cross section where each intersection of the second gridded cross section
15   including a "particular intersection" has an expected (mean) value of that parameter and a standard deviation; generate a Probability Density Function (PDF) from each expected value and standard deviation associated with each of the intersections of the grid, block 26;

20   (4) Integrate each Probability Density Function (PDF) to obtain a Cumulative Distribution Function (CDF) [recall that the area under the PDF curve is represented by the CDF], block 28;

     (5) Input a cutoff along the x-axis of the CDF and obtain a probability "Pcu" on the y-
25   axis, block 30;

     (6) Correct the probability "Pcu" by performing an "affine correction" on the probability Pcu, block 37; and

30   (7) In figure 3, assign the corrected value "(1-Pcu) [corrected]" to the "particular intersection" and assign a unique color to that corrected value, block 39 in figure 3.

The probability "(1-Pcu)" represents the probability that a parameter is greater than the cutoff. In our examples to follow, the "parameter" will be porosity; however, understand that the above method will work in connection with other types of parameters, including but not limited to porosity.

In addition, each lower cutoff "Xp" on the "lower cutoff map" (located at each intersection of a grid imposed on the map) is determined by performing the following additional method steps:

(8) In figure 2, input a 'probability' along the y-axis of the cumulative distribution function (CDF) and obtain a lower cutoff "Xp" on the x-axis of the CDF, block 32, figure 2;

(9) Correct the lower cutoff "Xp" by performing an "affine correction" on the lower cutoff "Xp", block 37; and

(10) In figure 3, assign the corrected value "Xp (corrected)" to the "particular intersection" and assign a unique color to that corrected value, block 41 in figure 3.

In addition, each "lower limit", "spread", and "upper limit" on three different types of "confidence limit maps" (located at each intersection of a grid imposed on the map) is determined by performing the following additional method steps:

(11) In figure 2, choose a "confidence level" on the y-axis of the CDF and obtain a "lower limit", a "spread", and an "upper limit" on the x-axis of the CDF, block 35;

(12) Correct the "lower limit", the "spread", and the "upper limit" by performing an "affine correction" on the "lower limit", the "spread" and the "upper limit", block 37;

(13) In figure 3, assign the corrected value "lower limit (corrected)" to the "particular intersection" and assign a unique color to that corrected value, block 43 in figure 3,

(14) assign the corrected value "spread (corrected)" to the "particular intersection" and assign a unique color to that corrected value, block 45 in figure 3, and

(15) assign the corrected value "upper limit (corrected)" to the "particular intersection" and assign a unique color to that corrected value, block 47 in figure 3.

Each of these steps will be discussed in detail with reference to figures 4 through 32.

Referring to figures 4 through 32, and referring initially to figure 4, block 22 of figure 3 indicates that, using a horizontal slice through a predetermined volume of earth including a plurality of wellbores, a cross section through that earth volume is generated, the cross section including a plurality of parameter data. One example of that parameter data is "porosity". However, it should be understood that the method and associated apparatus of the present invention, illustrated in figures 2 and 3, will also work in connection with other types of parameter data (other than porosity, which is being used by way of example only). The parameter data in the cross section represents the randomly scattered data samples 18 of figure 1. In figure 4, the predetermined volume of earth 34 has a cross section 36 passing therethrough. A plurality of wellbores 38 have been drilled in that volume of earth 34, the wellbores 38 passing through the cross section 36. In figure 5, one of those wellbores 38a is illustrated. The cross section 36 passes through the wellbore 38a of figure 5. A well logging tool 40 in the wellbore 38a generates a well log 42. The y-axis of that well log 42 represents units of a particular parameter (the x-axis being units of depth). In our example, we use the following parameter, but only by way of example for purposes of this discussion: porosity. Therefore, in figure 5, the intersection between the cross section 36 and the well log 42 represents a particular parameter value (e.g., porosity), as indicated by the "X" 44. As a result, in figure 4, the cross section 36 includes a plurality of parameter values 44a, 44b, 44c, and 44d (of porosity, which

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parameter is being used in our example for purposes of discussion only). In figure 6, that cross section 36 is illustrated again, and the plurality of parameter values (e.g., of porosity) 44a through 44d from figure 4 are located and illustrated in that cross section 36.

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In figure 6, the cross section 36 of figure 6 including the plurality of parameter values (e.g., of porosity) 44a through 44d in that cross section represent the "randomly scattered data samples" 18 of figure 1.

10 In figures 2 and 3, blocks 24, 26, 28, 30, 32, 35, 37, 39, 41, 43, 45, and 47 collectively comprise the "Uncertainty mapping software" 16 stored in the memory 16 of figure 1. Blocks 24, 26, 28, 30, 32, 35, 37, 39, 41, 43, 45, and 47 of figures 2 and 3 representing the Uncertainty Mapping Software 16 of figure 1 are discussed below with reference to figures 7 through 28 of the drawings.

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Recall block 24 of figure 2: "grid the cross section". Therefore, in figure 7, the first task at this point is to grid the cross section 46. In figure 7, grid lines 48 are drawn in the cross section 46, but the parameter values (of porosity in our example) 44a through 44d do not fall on an intersection of the grid 48. In accordance with block 24

20 of figure 2, the cross section 36 has been gridded as shown in figure 7.

Recall block 26 of figure 2: provide the gridded cross section including the plurality of parameter data as input data to a Kriging algorithm, execute the Kriging algorithm using that input data, and generate a second gridded cross section where each

25 intersection of the second gridded cross section including a "particular intersection" has an expected (mean) value of that parameter and a standard deviation; and generate a Probability Density Function (PDF) from each "expected value/standard deviation" associated with each of the intersections.

30 In figure 8, the gridded cross section of figure 7 including the plurality of parameter values (e.g., of porosity data) 44a through 44d are provided as input data to a Kriging

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Algorithm 50 (the Kriging algorithm has already been discussed in the above

paragraphs). The Kriging algorithm is executed using that input data, and, responsive

thereto, the Kriging algorithm generates a second gridded cross section 52 as shown

in figure 9, where each intersection of the second gridded cross section 52 (which

5 includes a particular intersection 54) has two values corresponding thereto: (1) an

expected (mean) value of the parameter (in our example, an expected value of

porosity), and (2) a standard deviation, block 56 of figure 9. That is, each intersection

54, 58, 60, 62,..., etc of the second gridded cross section 52 has its own "expected

value" of a parameter and its own "standard deviation" corresponding thereto. The

10 Kriging algorithm 50 performs a type of interpolation, using the parameter values

(of porosity in our example) 44a, 44b, 44c, and 44d, to determine a corresponding

expected (mean) value of a parameter at each intersection 54, 58, etc, and a

corresponding value of "standard deviation" at each intersection 54, 58, etc, of the

second gridded cross section 52 in figure 9. Since we are using porosity by way of

15 example only, at each intersection 54, 58, etc, of figure 9, a unique expected (mean)

value of porosity and a unique standard deviation will associated with each

intersection of the grid shown in figure 9.

In figure 10, let us examine, as an example, the intersection 54 of the second gridded

20 cross section 52 of figure 9. Intersection 54 now has two values of data associated

therewith: (1) a "particular expected (mean) value" of a parameter (such as "porosity"

in our example), where the expected value of the parameter is determined using an

interpolation technique performed by the Kriging algorithm 50 which utilizes the

randomly scattered parameter values (of porosity in our example) 44a-44d of figure 6,

25 and (2) a "particular standard deviation" (also determined by the Kriging algorithm

50), block 64 of figure 10.

In figure 10a, using the "particular expected (mean) value" and the "particular

standard deviation" which are both associated with intersection 54 and which were

30 both determined above with reference to figures 9 and 10, a "particular Probability

Density Function (PDF)" can now be determined, and that "particular Probability

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Density Function (PDF)" will now be associated with intersection 54. That "particular Probability Density Function (PDF)" is illustrated in figure 10a.

In figures 11, 12, and 13, the "particular Probability Density Function (PDF)" shown
5   in figure 10a, which is specifically associated with intersection 54 of the gridded cross section shown in figure 10, is illustrated again in figure 11 for purposes of this discussion. Recall block 28 of figure 2: "integrate each Probability Density Function (PDF) to obtain a "Cumulative Distribution Function". Therefore, in figure 11, integrate the Probability Density Function to thereby obtain the "Cumulative
10   Distribution Function" (CDF), block 66 of figure 12. That "Cumulative Distribution Function (CDF)" is illustrated in figure 13. However, since the "particular Probability Density Function (PDF)" of figures 10a and 11 is specifically associated with intersection 54 of the gridded cross section shown in figure 10, the "Cumulative Distribution Function (CDF)" of figure 13 is also specifically associated with
15   intersection 54 of the gridded cross section shown in figure 10.

Probability Map

A "probability map" is generated by plugging in the desired lower cutoff and
20   evaluating the probability "n" at any (x, y, z) location. That is, consider the following equations:

$$P_{100n}(xyz) = p\{\xi > co | xyz\} \tag{7}$$

$$xp = (co - \mu) / \sigma \tag{8}$$

25   $$n = F(xp) \tag{9}$$

where:

F = gaussian transform

Co = desired cutoff

30   P = calculated probability

20

σ = standard deviation

μ = mean

ξ = estimated variable

5   In figures 2 and 14, recall block 30 of figure 2: "Input a 'cutoff' along the x-axis of
the CDF and obtain a 'probability (1-Pcu)' on the y-axis". Therefore, in figure 14,
block 68, the user/operator inputs a 'cutoff' Xp along the x-axis of the CDF
(e.g., 10%) and obtains a probability "Pcu" on the y-axis. The probability "Pcu"
represents the probability that the parameter (e.g., porosity) is less than the cutoff
10   "Xp". Therefore, when the probability "Pcu" is determined, the probability
"(1-Pcu)" must represent the probability that the parameter is greater than the
cutoff "Xp".

In figure 15, for example, the user inputs a "cutoff" (e.g., "Xp") on the x-axis of the
15   Cumulative Distribution Function (CDF) and obtains a probability "Pcu" on the
y-axis of the Cumulative Distribution Function of figure 15. The probability "Pcu"
represents the probability that the parameter (e.g., porosity) is less than the cutoff
"Xp". In order to obtain the probability that the parameter is greater than the cutoff
"Xp", subtract "Pcu" from "1" to obtain "(1-Pcu)", as shown in figure 15.

20
In figures 15 and 16, recall that the CDF of figure 15 is specifically associated with
intersection 54 of the gridded cross section 52. In addition, recall that, for
intersection 54 of the gridded cross section 52 of figure 10, "(1-Pcu)" is the
probability that the parameter (such as porosity) is greater than the cutoff "Xp".

25
In figure 16, in order to plot one point on a specific "Probability map" which would
represent or illustrate the probability that a parameter, such as porosity, is greater than
a cutoff "Xp", assign and plot and superimpose onto intersection 54 of the gridded
cross section 52 the value "(1-Pcu)" and then assign a unique color to intersection 54
30   which would specifically identify that value "(1-Pcu)".

21

Lower Cutoff maps

Referring to figure 17, when producing Lower Cutoff maps, by transforming the
desired probability using the estimated local mean and standard deviation, the new
method and apparatus of the present invention will produce a map of lower cutoff
values. We can then say that, at any given (x, y) location, the "content" at that given
(x, y) location (i.e., the value of a parameter, such as porosity, at that intersection,
such as the intersection 54 of the gridded cross section 52) is at least the cutoff value
of the parameter with a probability of, for example, ninety percent (i.e., n = 0.9).
That is, for a lower cutoff ϕ and a given probability P_{100n}:

$$co(xyz) = xp \{p = .9 \mid xyz\} \tag{10}$$

$$xp = F^{-1}(p) \tag{11}$$

$$co = xp \cdot \sigma + \mu \tag{12}$$

where

F^{-1} = inverse gaussian transform

co = calculated cutoff

p = desired probability

σ = standard deviation

μ = mean

In figures 2, 17, and 18, recall block 32 of figure 2: Input a 'probability' along the
y-axis of the CDF and obtain a lower cutoff "Xp" on the x-axis of the CDF.
In figure 17, input a probability "Pcu" on the y-axis of the Cumulative Distribution
Function (CDF) and obtain a lower cutoff "Xp" on the x-axis of the CDF. In
figure 18, since the CDF of figure 17 is specifically associated with intersection 54 of
the gridded cross section 52, in order to plot one point on a specific "lower cutoff

22

map", assign and plot and superimpose onto intersection 54 the value "Xp" and assign a unique color which will specifically identify the value "Xp". As a result, we can say that, at intersection 54 of the gridded cross section 52 in figure 18, the probability is "Pcu" that the value of a parameter, such as porosity, at the intersection

5   54 is at least the cutoff value "Xp".

Confidence Limit maps

Referring to figure 19, when producing Confidence limit maps, a Confidence limit is

10 obtained (i.e., a lower limit, an upper limit, and a spread) by using "half the compliment of the confidence level" and "one minus this measure" (on the y-axis of the CDF) to thereby produce, on the x-axis of the CDF: (1) a 'lower limit' of the 'content' or 'value of a parameter', and (2) an 'upper limit' of the 'content' or 'value of the parameter'. The difference between these two lower and upper limits is the

15 data range (or 'spread') for the given confidence level. That is, for Lower and Upper

Bounds for P_{100n}:

$$xp_u = F^{-1}(p_u) \tag{13}$$

20 $$co_u = xp_u \cdot \sigma + \mu \tag{14}$$

$$xp_l = F^{-1}(p_l) \tag{15}$$

$$co_l = xp_l \cdot \sigma + \mu \tag{16}$$

25
$$\delta = xp_u - xp_l \tag{17}$$

where

30   p = confidence level

σ = standard deviation

μ = mean

35
xp_u = upper confidence limit

23

xp_l = lower confidence limit

co_u = upper cutoff

5

co_l = lower cutoff

δ = spread

10

In figure 2, recall block 35: choose a "confidence level on the y-axis of the CDF and obtain a "lower limit", a "spread", and an "upper limit" on the x-axis of the CDF.

In figure 19, for example, assume a given confidence level "p" of 90%, as illustrated

15   in figure 19. The confidence limits are chosen symmetrically about the 50% probability point on the vertical axis. For a confidence level of 90%, this results in an upper confidence limit of $50 + 90/2 = 95\%$ and a lower confidence limit of $50 - 90/2 = 5\%$. Refer now to figure 19, and note the upper and lower values on the y-axis of the Cumulative Distribution Function (CDF); that is, the lower value on the

20   y-axis of the CDF is 5% and the upper value on the y-axis of the CDF is 95%. Starting at the 5% value on the y-axis of the CDF of figure 19, intersect the CDF curve in figure 19 and note the "lower limit" on the x-axis of the CDF in figure 19. Similarly, starting at the 95% value on the y-axis of the CDF of figure 19, intersect the CDF curve in figure 19 and note the "upper limit" on the x-axis of the CDF curve

25   in figure 19. Furthermore, in figure 19, note that the difference between the "lower limit" and the "upper limit" on the x-axis of the CDF of figure 19 is called the "spread".

Referring to figure 20, the "lower limit" is plotted and assigned to intersection point

30   54, and a unique color is assigned to that "lower limit".

Referring to figure 21, the "spread" is plotted and assigned to intersection point 54, and a unique color is assigned to that "spread".

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Referring to figure 22, the "upper limit" is plotted and assigned to intersection point 54, and a unique color is assigned to that "upper limit".

In figure 2, recall the "affine correction" block 37.

5

Referring to figure 23, the values of '(1-Pcu)', 'Xp', 'lower limit', 'spread', and 'upper limit', which are assigned to intersection point 54 in figures 16, 18, 20, 21, and 22, respectively, must now undergo an "affine correction". The "affine correction" will be discussed in detail below with reference to figure 29; however, suffice it to say

10   at this point that the mean and the standard deviation associated with each of the original data (in figure 6) and each of the "Kriged" data (in figure 9) are not the same. That is, after Kriging (figure 8), the mean and standard deviation of each of the Kriged data (figure 9) is less than the mean and standard deviation of each of the original data (figures 6 and 7). Therefore, in order to remedy this problem, the

15   values of '(1-Pcu)', 'Xp', 'lower limit', 'spread', and 'upper limit' must undergo the "affine correction", prior to plotting and assigning such values to the intersection point 54 of the gridded cross section 52, in order that the mean and standard deviation of each of the Kriged data after undergoing the "affine correction" (figure 23) is equal to the mean and standard deviation of each of the original data

20   (figures 6 and 7). In figure 23, therefore, each of the values '(1-Pcu)', 'Xp', 'lower limit', 'spread', and 'upper limit' undergo the "affine correction", block 70 of figure 23, thereby generating the following corrected values: '(1-Pcu)[corrected]', 'Xp(corrected)', 'lower limit (corrected)', 'spread (corrected)', and 'upper limit (corrected)'.

25

In figure 2, recall blocks 39, 41, 43, 45, and 47, wherein the corrected values '(1-Pcu)[corrected]', 'Xp(corrected)', 'lower limit (corrected)', 'spread (corrected)', and 'upper limit (corrected)' are assigned to the particular intersection and a unique color is assigned to that corrected value.

30

25

Referring to figure 24, the corrected value "(1-Pcu)[corrected]" is plotted and assigned to intersection point 54, and a unique color is assigned to that corrected value.

5   Referring to figure 25, the corrected value "Xp(corrected)" is plotted and assigned to intersection point 54, and a unique color is assigned to that corrected value.

Referring to figure 26, the corrected value "lower limit (corrected)" is plotted and assigned to intersection point 54, and a unique color is assigned to that corrected
10   value.

Referring to figure 27, the corrected value "spread (corrected)" is plotted and assigned to intersection point 54, and a unique color is assigned to that corrected value.

15   Referring to figure 28, the corrected value "upper limit (corrected)" is plotted and assigned to intersection point 54, and a unique color is assigned to that corrected value.

Referring to figure 29, regarding the "affine correction", in order to compensate for
20   the smoothing effect of Kriging on the estimated mean and variance, an "affine correction" is applied to each value [such as '(1-Pcu)' and 'Xp' and 'lower limit' and 'spread' and 'upper limit'] that is assigned to each intersection (such as intersection 54) of the gridded cross section 52 shown in figure 9 (and figures 16, 18, 20, 21, and 22). This is accomplished by co-generating the Kriged estimation grid. The mean
25   and standard deviation of this Kriged estimation grid are then calculated and compared with the mean and standard deviation of the data. The correction ensures that the resulting maps are derived from the same global mean and standard deviation as the original data.

30   In order to Compensate for the smoothing effect for the lower cutoff map:

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$$xp = F^{-1}(p) \tag{18}$$

$$co = xp \cdot \sigma_1 + \mu_1 \tag{19}$$

$$co' = (\sigma'/\sigma)(co-\mu) + \mu' \tag{20}$$

where:

F^{-1} = inverse gaussian transform

co = calculated cutoff

p = desired probability

σ' = standard deviation of the original data

σ = standard deviation of the co-generated realization

μ' = mean of the original data

μ = mean of the co-generated realization

co' = corrected cutoff

In order to Compensate for the smoothing effect for the probability map:

$$co' = (\sigma/\sigma')(co - \mu') + \mu \tag{21}$$

$$xp = (co' - \mu_1)/\sigma_1 \tag{22}$$

$$p' = F(xp) \tag{23}$$

where:

F = gaussian transform

co = desired cutoff

σ' = standard deviation of the original data

σ = standard deviation of the co-generated realization

μ' = mean of the original data

μ = mean of the co-generated realization

co' = corrected cutoff

5   p' = corrected probability

In figure 29, for example, when the original data 36 (from figure 6 and 7), having a mean X and a standard deviation Y, are Kriged, via the Kriging Algorithm 50, the resultant Kriged data 52 is produced, where the resultant Kriged data 52 has a mean A

10   and a standard deviation B, and where A < X and B < Y. However, when the Kriged data undergoes the affine correction 70, the final result 72 has a mean X and a standard deviation Y. The corrected values of '(1-Pcu)[corrected]', 'Xp(corrected)', 'lower limit (corrected)', 'spread (corrected)', and 'upper limit (corrected)' correspond to the final result 72.

15

Referring to figures 29, 30, 31, and 32, an example of a Probability Density Function representing the original data 36 of figure 29 is shown in figure 30 and note that the mean is 4.3504 and the standard deviation is 6.7026. However, an example of a Probability Density Function representing the Kriged data 52 of figure 29 is shown in

20   figure 31 and note that the mean is 2.5077 and the standard deviation is 2.9313 (which is less than the mean of 4.3504 and the standard deviation of 6.7026 of the original data 36 shown in figure 30). On the other hand, an example of a Probability Density Function representing the Final Result 72 of figure 29, after the affine correction 70 has been applied to the Kriged data 52, is shown in figure 32 and note

25   that the mean is 4.3504 and the standard deviation is 6.7026 (which is equal to the mean of 4.3504 and the standard deviation of 6.7026 of the original data 36 shown in figure 30).

Referring to figures 33, 34, and 35, these figures will illustrate some of the above

30   features.

Figure 33, which includes figures 33a, 33b, 33c, and 33d, illustrates (in figure 33a) a

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"one-out-of-a hundred" realization generated with conditional simulation. Figure 33c shows a lower cutoff porosity map with a probability of .9 illustrating that the actual porosity will be higher. Figure 33d is the probability map for a user defined 10% lower cutoff porosity. Figure 33b is a spread in porosity for a confidence level of

5 90%.

Figure 34, which includes figures 34a, 34b, 34c, and 34d, shows probability maps for different porosity cutoffs. As one would expect, the probability to find higher porosity cutoffs gets progressively smaller.

10

Figure 35, which includes figures 35a, 35b, 35c, and 35d, shows in cross section the effect which the exactitude principle has on the smoothness of the resulting surface as well on the error variance. Note that the examples are produced in two-dimensional maps, but the method is formulated as a general 3D solution.

15

In figure 4, recall that a cross section 36 through a cubic volume of earth formation 34, which includes a plurality of random variables (scattered data samples) 44a, 44b, 44c, and 44d, can be mapped; that is, a map can be generated which represents that cross section 36, such as the maps discussed above with reference to figures 16, 18,

20 20-22, and 24-28.

Referring to figure 36, the cubic volume of earth formation 34 of figure 4 is illustrated again in figure 36. However, in figure 36, the cubic volume of earth formation 34 includes three cross sections (provided as an example for purposes of this discussion

25 only): cross section 36a, cross section 36b, and cross section 36c. Each cross section 36a through 36c includes the intersection 54 of figures 9 and 10. In addition, each cross section 36a-36c includes the plurality of parameter values 44a through 44d. The above discussion with reference to figures 1 through 29 disclosed how a 'corrected value' is assigned to the intersection 54 and a unique color is assigned to

30 that corrected value. For example, in figure 24, the corrected value

"(1-Pcu)[corrected]" was assigned to intersection 54 and a unique color was assigned thereto. Similarly, in figures 25, 26, 27, and 28, the corrected values "Xp(corrected)", "lower limit(corrected)", "spread(corrected)", and "upper limit(corrected)" were assigned to intersection 54 and unique colors were assigned thereto. In figure 9,

5 using the novel methods discussed above, other such 'corrected values' [i.e., (1-Pcu)[corrected], Xp(corrected), lower limit(corrected), spread(corrected), and upper limit(corrected)] are also assigned to all the other intersections, such as intersections 58 and 60 and 62, of the gridded cross section 52 of figure 9. As a result, in accordance with one aspect of the present invention, the maps of figures 33,

10 34, and 35 were generated.

However, since the cubic volume of earth 34 of figure 36 comprises a plurality time slices/horizons or cross sections 36a-36c, and since the novel methods of the present invention can generate a map for each such cross section, the novel methods

15 discussed above which are practiced by the present invention can also be used to generate a cube 34, such as the cube 34 of figure 43. A method for generating a cube 34 is discussed below with reference to figures 37 through 43 of the drawings.

In figure 37, the novel methods discussed above for generating the 'corrected values'

20 illustrated in figure 23 {i.e., '(1-Pcu)[corrected]', 'Xp(corrected)', 'lower limit(corrected)', 'spread(corrected)', and 'upper limit(corrected)'} are used to determine three 'corrected values' (i.e., corrected value A, corrected value B, and corrected value C) for assignment, respectively, to intersections 80, 82, and 84, as shown in figure 37.

25 In figure 38, the novel methods discussed above for generating the 'corrected values' are used to determine three additional 'corrected values' D, E, and F for assignment, respectively, to intersections 86, 88, and 90, as shown in figure 38.

In figure 39, the novel methods discussed above for generating the 'corrected values'

30 are used to determine three additional 'corrected values' G, H, and I for assignment, respectively, to intersections 92, 94, and 96, as shown in figure 39.

In figure 40, the novel methods discussed above for generating the 'corrected values' are used to determine three additional 'corrected values' J, K, and L for assignment, respectively, to intersections 98, 100, and 102, as shown in figure 40.

5

In figure 41, the novel methods discussed above for generating the 'corrected values' are used to determine three additional 'corrected values' M, N, and O for assignment, respectively, to intersections 104, 106, and 108, as shown in figure 41.

10 In figure 42, the novel methods discussed above for generating the 'corrected values' are used to determine three additional 'corrected values' P, Q, and R for assignment, respectively, to intersections 110, 112, and 114, as shown in figure 42.

In figure 43, when all the intersections on each cross section 36a, 36b, and 36c of the
15 cube 34 have a 'corrected value' assigned thereto, the cube 34 of figure 43 will be generated. When unique colors are assigned to each such 'corrected value', a three dimensional (3D) cube, representing the cubic volume of earth 34 shown in figure 4, will be generated.

20 The invention being thus described, it will be obvious that the same may be varied in many ways. Such variations are not to be regarded as a departure from the spirit and scope of the invention, and all such modifications as would be obvious to one skilled in the art are intended to be included within the scope of the following claims.